

DESIGN OF FOUNDATIONS ON BLACK COTTON SOIL BASED ON RELIABILITY

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Abstract - The basic requirements in the conventional designs are the ultimate limit state (ULS), serviceability limit state (SLS), reliability and economic requirements. Current design practice usually addresses the ULS and SLS requirements explicitly using a trial-and-error approach, in which a trial design is proposed and is checked against the ULS and SLS requirements. Although there have been attempts to develop reliability based design of foundation in cohesive soil, the same factoring in the spatial variability of parameters in black cotton soil has been hitherto not comprehensively analysed. This study besides attempting to develop an RBD using first order reliability methods, also aims to analyse the effect of variation of geotechnical parameters on the reinforcement requirements for an isolated square shallow foundation in black cotton soil.

Keywords: first-order reliability methods (FORM), Reliability based design, Black cotton soil, AFOSM, limit state.

1. Introduction

The purpose of the foundation is to transmit the loads from the structure which to the soil without any shear failure and limit settlement of the structure within permissible limits. Thus, the design of the foundation must confirm to these requirements. The foundations are generally classified into shallow and deep. They differ in terms of geometry, the behaviour of the soil, its constructive systems and its structural functionality. A shallow foundation can be broadly defined as a structural member whose cross section is of large dimensions with respect to its depth below the ground surface and hence it transfers the loads of a superstructure at relatively lower depths. Constructive systems for shallow foundations generally do not involve any significant complications. Shallow foundations are further classified into isolated footing, combined footing, strip footing, or mat foundation.

1.1 Reliability Analysis

It is now a known fact that uncertainties are unavoidable in geotechnical engineering and that it becomes imperative to quantify these uncertainties. (e.g., Casagrande 1965; Whitman 2000; Phoon and Kulhawy 1999a,b). These uncertainties can be attributed to variations in loads, geologic site interpretations, geotechnical properties, computational models, etc. The

quantification of the uncertainties associated with geotechnical property characterization is an indispensable requirement. The foundation design scheme is negatively affected by a lack of adequate characterization of sub-surface conditions and scarcity of relevant information, which inevitably leads to in design uncertainty. Therefore, a need arises for assessing uncertainties and their likely impacts on engineering designs that can be interpreted and implemented readily

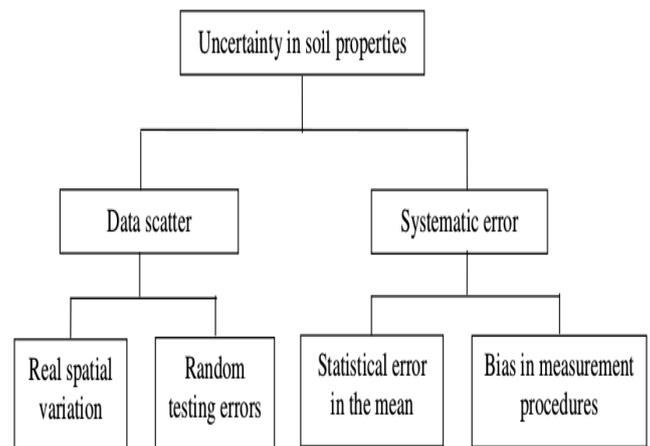


Fig.:-Categories of sources uncertainty in soil properties (Whitman 2000)

Design is a process of determining a set of parameters that describe and specify a structure (e.g. isolated foundation) which satisfies a series of performance requirements. When it comes to reliability-based analysis these performance requirements can be specified in terms of a pre-defined target probability of failure (pf) or reliability index (β).

The limit state function is defined as a function of capacity and demand; it is denoted as g and expressed

$$g(R, Q) = R - Q \text{ Eq. 1}$$

Where: R - is the structural resistance or capacity of the structural component and Q is the load effect or demand of the structural component with the same units as the resistance. The performance function $g(X)$ is a function of capacity and demand variables (X_1, X_2, \dots, X_n) which are basic random variables for both R and Q) such that,

$$\begin{aligned}
 g(X_1, X_2, \dots, X_n) > 0 & \text{ safestate} \\
 & = 0 \text{ limit state} \\
 & < 0 \text{ failurestate}
 \end{aligned}$$

Where: $g(X) = 0$ is known as a limit state surface and each X indicates the basic load or resistance variable. The probability of failure (P_f) can be related to an indicator called the reliability index, β .

For the estimation of the probability of failure, the method employed involves approximate iterative calculation procedures. In this method, two important measures are used:

Expectations: $\mu_i = E []$, $i = 1, 2, \dots, n$ Eq.2

Covariances: $C_{ij} = Cov [,]$, $i, j, = 1, 2, \dots, n$ Eq.3

The safety margin is the random variable $M = g(x)$ (also called the state function). Non-normal variables are transformed into independent standard normal variables, by locating the most likely failure point, β -index (called the reliability index), through an optimization procedure. This is also done by linearizing the limit state function in that point and by estimating the failure probability using the standard normal integral. The reliability index β , is then defined by Hasofer and Lind as,

$$\beta = \frac{\mu_m}{\sigma_m} \text{ Eq. 4}$$

Where: μ_m - mean of M and σ_m - standard deviation of M .

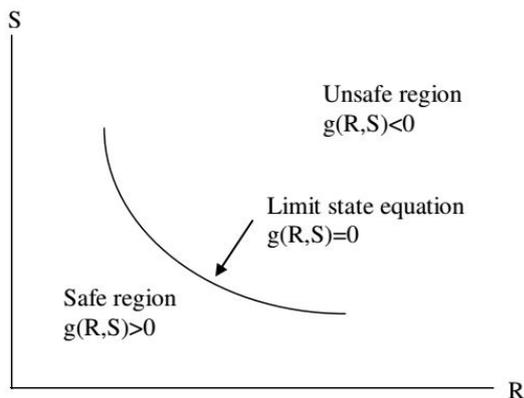


Fig:- Limit state showing failure surface, safe and unsafe regions

If R and S are uncorrelated and with $M = R - S$

Then,

$$\mu_m = \mu_R - \mu_S$$

$$\sigma_m^2 = \sigma_R^2 + \sigma_S^2$$

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}}$$

1.2 Objectives of the this Study

1. Conduct a reliability analysis to determine the effect of variation of various parameters on target reliability index and hence find corresponding optimum factor of safety.

2. Using the allowable bearing pressure computed as mentioned above to get the reinforcement requirements.

3. Illustrate through a design example the reliability-based analysis for design of shallow foundations.

4. Develop RBD charts establishing the effect of various factors on area of steel required.

2. Methodology

This study applies the Hosafer-Lind reliability index to the computation of bearing capacity of soil. The shear failure criterion and settlement criterion are taken into consideration. Based on the values obtained from deterministic and probabilistic analysis the foundation considered is design using IS code 6403-1981 and the results are compared. A general description of the method is illustrated here

STEP 1: Moment is computed using allowable bearing pressure computed in step 1 for a given target reliability index or probability of failure.

STEP 2: Thickness of footing is computed based on one-way shear and two-way shear criterion.

STEP 3: Compute area of steel required as

$$M_u = 0.87 f_y A_{st} d \{1 - (A_{st} f_y / f_{ck} b d)\} \text{ Eq. 21}$$

$$f_y = 415 \text{ N/mm}^2, f_{ck} = 20 \text{ N/mm}^2$$

STEP 4: Check whether the area of steel computed is consistent with the requirements for step 2. If not return to step 2 and redesign.

3. Results And Discussions

The soil medium is considered to be elastic homogeneous semi-infinite. The soil is considered to be a $c-\phi$ soil. The drained cohesion, angle of internal friction and unit weight of soil is treated as independent variables. Although there is a tendency to model black cotton soil as purely cohesive soils by designers for conservatism, it is only reasonable to take a lower value of ϕ based on relevant codes and/or previous studies. For the standard case mean values for c , ϕ and γ are taken as 20 KPa, 20° and 18 kN/m³ and a coefficient of variation (COV) of 0.1 is considered for all three variables.

The load from the column is taken as 400 kN and column dimension in plan is 500 mm x 500 mm. The load is assumed to be vertical and acting at the centre. Therefore, effect of eccentricity and inclination has not been considered.

The deterministic bearing capacity is computed using the formulation in IS 6403-1981. A factor of safety of 2.5 is assumed which is in consonance with current design practices. Considering the standard case of 1m x 1m footing the result of the analysis has been described in subsequent sections.

3.1 Effect of reliability Index.

Different values for probabilistic bearing capacity for foundations for specified conditions were

computed for various reliability index. The probabilistic factor of safety is computed as the ratio of actual deterministic bearing capacity (without dividing it by deterministic safety factor) to the probabilistic bearing capacity. As anticipated the FOS decreases with increasing reliability index. It is worth mentioning that the reliability index is mentioned as ratios. So a reliability index of 0.999 corresponds to 99.9% reliability or 0.1% probability of failure. The results are represented in a tabular form.

Table :- Variation of FOS with respect to reliability index.

S.no.	Reliability Index	F.O.S
1	0.9999	1.78
2	0.999	1.62
3	0.99	1.45
4	0.95	1.3
5	0.9	1.23

This indicates that even at a very low probability of failure of 0.01% the FOS required is much less than the one used in the conventional design suggesting that they are highly conservative.

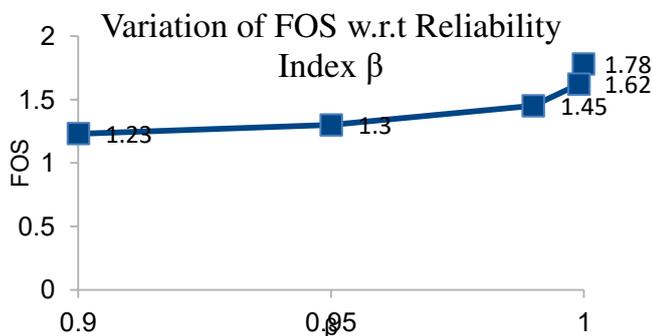


Fig:- Variation of FOS with respect to reliability index

This effect can be represented in another manner if we plot FOS vs probability of failure in semi-log plot.

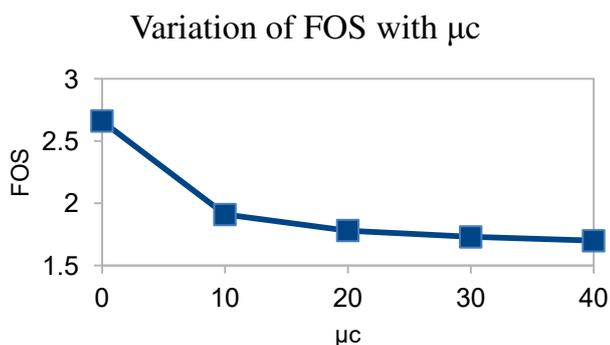


Fig:- FOS vs Probability of failure (%)

3.2 Effect of angle of internal friction, ϕ

The effect of variation of mean and standard deviation values for a specified reliability index (0.9999) has been investigated. The values are varied from 0 to 30 degrees to analyse the impact over a much wider range.

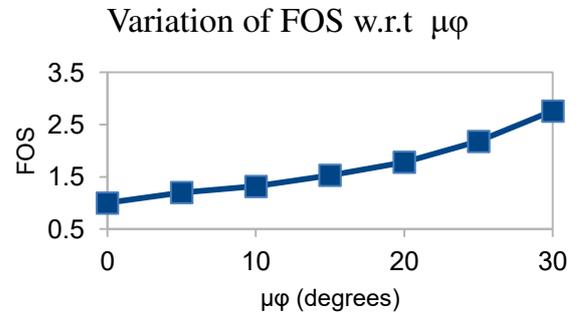


Fig:- Variation of FOS with μ_ϕ

It can be observed that FOS monotonously increases with the mean value of ϕ (μ_ϕ). An interesting finding is that as the value of ϕ increases the choice of using higher value of FOS is reasonable. This can be attributed to the fact that the effect of ϕ is profound and any uncertainty in its determination will lead to unsafe design if appropriate FOS is not chosen. The detailed explanation of this effect is beyond the scope of this discussion.

Another key observation is that as the value of ϕ approaches zero the probabilistic bearing capacity approaches actual deterministic bearing capacity which further emphasizes the point that choice of a much lower factor of safety is justified. It must however be understood that pure cohesive soil is only an ideal scenario and not an actual pragmatic consideration.

Variation of FOS w.r.t μ_ϕ for different cov

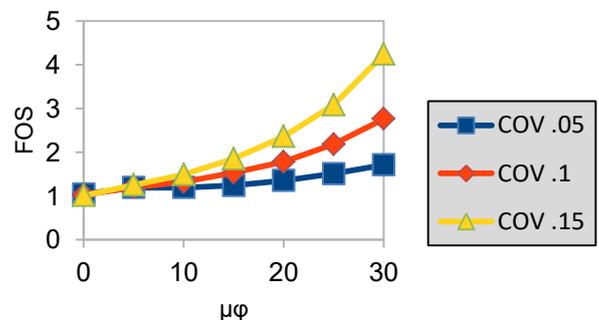


Fig:- Variation of FOS with μ_ϕ for different σ_ϕ

Fig shows the trends in FOS for three different values of standard deviation chosen for the ϕ . It suggests that for the initial phase (up to 5 degrees) the values are

fairly close to one another. But as higher values are approached there is considerable deviation. The curve showing the variation of FOS for $COV=0.15$ reaches values as high as 4.3 for $\phi=30$ degrees. This suggests that designing a foundation with presumed values for soils with ϕ values greater than 20 degrees can lead to reasonably unsafe designs.

3.3 Effect of cohesion, μ_c .

The mean value of cohesion or μ_c has been varied from 0 to 40. The value of σ_c has been taken as 0.1 times μ_c for the standard case. This further lead credence to the fact that for a frictional soil or an ideal cohesionless soil the requirement of higher FOS is only logical. As the value of c increases FOS decreases sharply in the beginning and the slope gradually flattens. The value for FOS reaches at 1.7 for a value of cohesion as 40 kPa.

Variation of FOS with μ_c

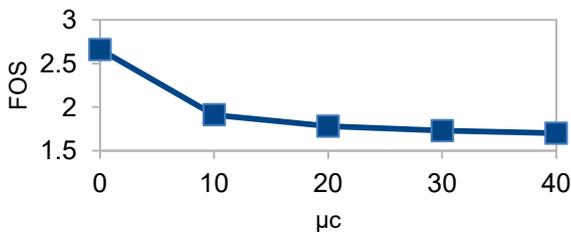


Fig:- Variation of FOS with μ_c

The FOS can be observed to be insensitive to the variation in value of standard deviation of cohesion, σ_c . Table conclusively demonstrates this assertion.

Table: - Variation of FOS μ_c for different σ_c

S.No.	μ_c	σ_c equals $0.05\mu_c$	σ_c equals $0.1\mu_c$	σ_c equals $0.15\mu_c$
1	12	1.01	1.01	1.01
2	14	1.12	1.12	1.12
3	16	1.22	1.22	1.22
4	18	1.33	1.33	1.32
5	20	1.43	1.43	1.43
6	22	1.53	1.53	1.53
7	24	1.64	1.64	1.63

3.4 Effect of unit weight, γ .

The mean value of cohesion or μ_γ has been varied from 15 kN/m³ to 20 kN/m³. The value of σ_γ has been taken as 0.1 times μ_γ for the standard case. As the value of γ increases, FOS initially increases plateaus in a region of 16-17 and then further increases from there. The value for FOS reaches at 1.8 for a value of unit weight as 20 kN/m³.

Variation of FOS w.r.t γ_{mean}

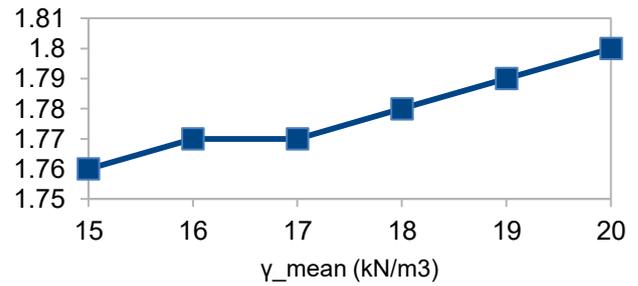


Fig:- Variation of FOS with μ_γ

It can also be seen in the table below that FOS is insensitive to the variation in σ_γ . Thus, it can be safely presumed that within the normal range of unit weight, uncertainty in its determination does not bring about appreciable effect in design of foundation.

Table:- Variation of FOS μ_c for different σ_c

S.No.	μ_c	σ_c equals $0.05\mu_c$	σ_c equals $0.1\mu_c$	σ_c equals $0.15\mu_c$
1	16	1.39	1.39	1.39
2	17	1.41	1.41	1.41
3	18	1.43	1.43	1.43
4	19	1.45	1.45	1.45
5	20	1.47	1.47	1.47
6	21	1.49	1.49	1.49

4 Design Example based on soil properties determined from soil tests.

The soil sample is collected from a location near Hathaikheda Dam, Anand Nagar Bhopal. The soil sample is collected from a depth of 1m. The colour of the soil is greyish. Visual identification of the soil reveals that it is clay/silty clay.

The following tests are conducted for determination of soil properties.

1. MC determination by oven drying method.
2. Bulk Density determination by core cutter
3. Sieve Analysis
4. Liquid limit and plastic limit tests.
5. UU triaxial test.
6. Unconfined Compression test.

4.1 Moisture content Determination

Oven drying method is used for moisture content determination in accordance with IS:2720Part II S-1.

Table :- Moisture Content Determination

S.No.	Weight of container (gm)	Weight of container moist soil	Weight of container dry soil	Moisture content
1	16	69	59.7	21.28
2	14.5	68.3	57.8	24.25
3	16.5	74	62.4	25.27

The average moisture content is 23.6%.

4.2 Determination of bulk density

The core cutter test is conducted in accordance with IS 2720 Part XXIX. The in-situ bulk density of soil determined using core cutter method is 2.01 g/cm³ whereas the dry density considering the moisture content as determined in the previous section is 1.62 g/cm³

Table :- Bulk and dry density of soil

Weight of core cutter (gm)	Volume of core cutter	Weight of core cutter soil (gm)	Bulk Density	Dry density
840	1000	2849.62	2.01	1.62

4.3 Classification of soil

Classification of soil is done in accordance with IS 1498. The process involves grain size distribution and plasticity analysis. Grain size distribution is done using standard IS sieves.

Grain Size Distribution

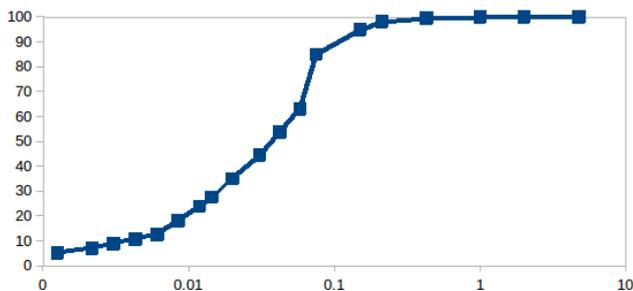


Fig:- Grain size Distribution curve.

Gravel = 0 % , Sand =13.76% Silt and Clay =86.24%
 Since more than 50% particles pass through .075 mm sieve hence the soil can be classified under fine

grained soil. The grain size distribution graph reveals that the soil is a well graded soil

4.4 Atterberg's Limits

Liquid Limit =31, Plastic Limit = 20.3,
 Plasticity Index = 10.7

This in the region of low plasticity above the A-line in the plasticity chart (IS 1498 3.5.3, Fig 1)Hence the soil is classified as CL (low plasticity clay)

4.5 Triaxial Tests

Triaxial tests were conducted to determine the undrained cohesion and friction of soil as well as the **unconfined compressive strength** of soil. Three cylindrical samples of size 76 cm length and 38 cm diameter were tested under confining pressures of 100, 200 and 300 kPa.

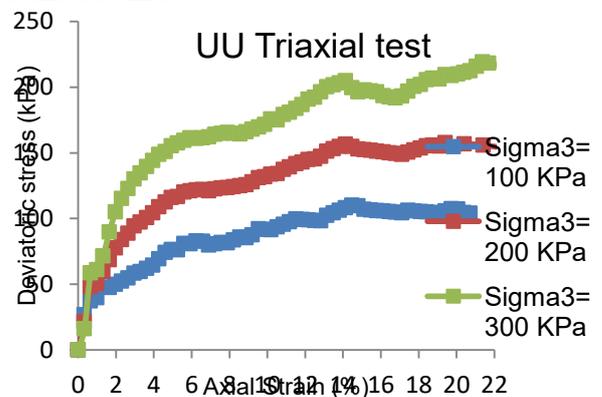


Fig:- Deviatoric stress vs Axial strain for UU triaxial tests

4.6 Unconfined Compression test.

The results of peak deviatoric stress and mean stress at failure is plotted in p-q plain to determine cohesion and angle of internal friction of soil.

Angle of internal friction(Φ) =12.50
 Cohesion (c) =21.2 kPa

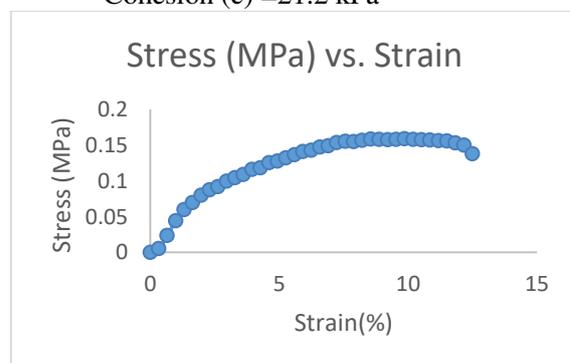


Fig:- unconfined compressive strength test results in form of stress vs strain

The peak stress at failure or unconfined compressive strength is computed to be 158.6 kPa. Therefore, the value of c_{uis} is 79.3 kPa. This value is used for the settlement computation.

5 Comparison between Reliability based Design and Deterministic Design

The mean values of cohesion, angle of internal friction and unit weight for the design are taken to be as determined from the tests conducted. The footing is considered to be at a depth of 1m. Reliability Index of 0.9999 is considered. The permissible settlement has been conservatively set to 25 mm. The transverse reinforcement is assumed to be equal to the longitudinal one.

Angle of internal friction (Φ) = 12.50

Cohesion (c) = 21.2 kPa

Unit weight (γ) = 19.2 kN/m³

Safe Bearing capacity Based on Shear Failure = 274.6 kPa

Safe Bearing capacity Based on Settlement = 292.6 kPa

Footing width = 1.27 m.

Thus, we choose the lower value 274.6 kPa.

The value is adjusted for the required footing width.

Required overall depth = 254 mm

Area of steel required = 471 mm²

Computations for the deterministic design are mentioned below.

Safe Bearing capacity Based on Shear Failure = 152.6 kPa

Safe Bearing capacity Based on Settlement = 218.3 kPa

Footing width = 1.7 m.

Thus, we choose the lower value 152.6 kPa.

The value is adjusted for the required footing width.

Required overall depth = 326 mm

Area of steel required = 641 mm²

Thus, we can see that there is a significant reduction in the percentage of steel and volume of concrete if reliability-based design is considered.

6. CONCLUSIONS

The necessity of economic optimization limit state (EOLS), alongside the requirements of ultimate limit state (ULS) and serviceability limit state (SLS), are three basic requirements in foundation designs. This thesis explores the economic optimisation design procedure to take into account the geotechnical uncertainties and combines reliability-based methodologies together with the economic optimization design framework to measure the likely impact of geotechnical uncertainties on foundation construction expenses to black cotton soils. A reliability-based economical design optimization framework is developed initially, followed by a good example that is applicable to the design of a isolated foundation under drained loading. Parametric analysis is conducted to study the impact of variables in geotechnical design. Then effects of geotechnical property uncertainties on optimized foundation construction cost will be explored in terms of steel and concrete requirements. The salient features of the analysis and design performed in the analysis are highlighted below.

1. It can be observed that FOS monotonously increases with the mean value of ϕ (μ_ϕ). An interesting finding is that as the value of ϕ increases the choice of using higher value of FOS is reasonable. This can be attributed to the fact that the effect of ϕ is profound and any uncertainty in its determination will lead to unsafe design if appropriate FOS is not chosen.

2. As the value of c increases FOS decreases sharply in the beginning and the slope gradually flattens. The value for FOS reaches at 1.7 for a value of cohesion as 40 kPa.

3. As the value of γ increases, FOS initially increases plateaus in a region of 16-17 kN/m³ and then further increases from there.

4. Significant reduction in the volume of concrete required for construction of footing can be achieved by employing reliability analysis. A reduction of 39.67 is observed for a failure probability of 0.01%.

5. Reduction in the steel required (17.69) is considerably high considering the impact on overall cost of the design.

6. The conventional design results using FOS value as 2.5, are almost similar to reliability-based design for COV=0.15. If we compute the corresponding FOS for this analysis it comes out to be about 2.37.

It can be safely concluded that geotechnical analysis of soil is an imperative and indispensable. In absence of soil data, no amount of speculation can either ensure safety or economic conservativeness. Although factor of safety approach is simple and straightforward, it does not consider different sources of uncertainty in geotechnical design in a rational manner. In order to incorporate these variations, reliability analysis is performed. In this approach, input soil parameters are treated as random variables and the influence of these input random variables on the output random variable is studied. Reliability analysis approaches can be used in conjunction with conventional approaches to have better insight into the choice of allowable value of bearing pressure and helps in decision-making process.

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